KINETIC STUDY OF HIGH-FREQUENCY INSTABILITY IN GAS DISCHARGES

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It is well known [1] that instability can develop in a gas-discharge plasma because of the presence of a decreasing segment in the dependence of electron drift velocity on the applied electric field. This instability is analogous to the Gunn effect in semiconductors [2]. The linear stage of development of such instability was studied in [1, 3, 4]. Depending on the value of the parameter $\nu_u\tau_m = E^2/(4\pi n_e kT_e)$, the rate of instability development is determined either by the frequency τ_m^{-1} (at $\tau_m\nu_u \gg 1$), or by ν_u ($\tau_m\nu_u \ll 1$) [3] (where ν_u is the inelastic collision frequency, which defines the relaxation rate of the symmetrical component of the electron distribution function, $\tau_m = 1/(4\pi\sigma)$, σ is the plasma conductivity, E is the electric field intensity, n_e is the electron concentration, and T_e is the mean electron energy). In the second case instability development is not described by the transfer equations, and it is necessary to calculate the electron distribution function. To analyze the role played by kinetic effects, we shall consider homogeneous development of Gunn-type instability, using model inelastic collision integrals. Self-similar solutions of the kinetic equation will be found for constant current in the circuit. Numerical solution of the kinetic equation will show that the full solution over time periods of the order of ν_u^{-1} approaches self-similarity.

In the case considered, the problem reduces to solution of the spherically symmetric portion of the distribution function f_0, which in a spatially homogeneous plasma has the form

$$\frac{\partial f_0}{\partial t} = \frac{e^2E^2}{3m^2}\frac{1}{v^2}\frac{\partial}{\partial v}\frac{v^2}{\nu_m}\frac{\partial f_0}{\partial v} + \mathrm{St}\, f_0 = 0, \tag{1}$$

where v is the modulus of the electron velocity, ν_m is the elastic collision frequency, $\mathrm{St} f_0$ is the inelastic collision integral, and m is the mass of the electron. Since we are concerned with rapid processes with characteristic frequencies of the order of ν_u, the ion concentration can be considered frozen, and given the condition $\tau_m\nu_u \ll 1$, which we assume fulfilled, we may neglect the change in electron concentration, which to good accuracy is equal to the ion concentration. The condition for normalization of the distribution function then has the form

$$\int_0^\infty v^2 f_0 dv = 1. \tag{2}$$

With the assumptions made, given circuit conditions which maintain the current constant, the current constancy condition can be written in the form

$$W_e = -\frac{eE}{3m}\int_0^\infty \frac{v^3}{\nu m}\frac{\partial f_0}{\partial v}dv = \text{const}. \tag{3}$$

Thus, the problem of Gunn instability development under the conditions specified reduces to solution of Eqs. (1)-(3).

Instability development can be explained qualitatively in the following manner. For ex-

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ample, assume a field fluctuation develops such that the field increases. The distribution function begins to relax to a new form corresponding to the new field value. The value of the integral in Eq. (3) must then decrease, since the electron drift velocity decreases with increase in field. At W_e = const the decrease in mobility leads to a further increase in field. The distribution function then spreads out along the velocity axis. Linear theory predicts an aperiodic development of the instability, the direction of which is determined by the initial direction of the field fluctuation toward increase or decrease.

We then choose the collision integral in the divergent form

$$\mathrm{St}\, f_0 = \frac{1}{v^2}\frac{\partial}{\partial v}(\nu_u v^3 f_0). \tag{4}$$

We write the velocity dependences of ν_m and ν_u in model form

$$\nu_m = \nu_0 v^p,\ \nu_u = \delta\nu_m,\ \delta = \delta_0 v^q, \tag{5}$$

where p, q are real numbers. We note that $\mathrm{St}f_0$ has a divergent form when the threshold for inelastic processes is much less than T_e. A power series representation of the frequency as a function of velocity is valid only over a certain energy interval. For the Gunn effect in gases, this model is purely qualitative, while for semiconductors it is apparently applicable over a wider range. In the special case where p = q, this integral describes energy losses in elastic collisions. With consideration of Eqs. (4), (5), Eqs. (1), (3) take on the forms

$$\frac{\partial f_0}{\partial t} = \frac{e^2E^2}{3m\nu_0 v^2}\frac{\partial}{\partial v}v^{2-p}\frac{\partial f_0}{\partial v} + \frac{\nu_0\delta_0}{v^2}\frac{\partial}{\partial v}v^{3+q+p}f_0; \tag{6}$$

$$-\frac{eE}{3m\nu_0}\int\limits_0^\infty v^{3-p}\frac{\partial}{\partial v}f_0 dv = W_e = \mathrm{const}. \tag{7}$$

In the stationary case the distribution function and corresponding drift velocity can be found easily:

$$f_{00} = \frac{s}{\alpha^3\Gamma\left(\frac{3}{s}\right)}\exp\left[-\left(\frac{v}{\alpha}\right)^s\right],\quad W_{e0} = \frac{eE}{3m\nu_m(\alpha)}\Gamma\left(\frac{3-p}{s}\right)/\Gamma\left(\frac{3}{s}\right),$$

$$\alpha = \left[\frac{s}{3\delta_0}\left(\frac{eE}{m\nu_0}\right)^2\right]^{\frac{1}{s}},\quad s = 2p+q+2>0.$$

From the expressions presented it is evident that $W_{e0} \sim E^{(q+2)/s}$, and consequently, the field dependence of drift velocity will be falling when the condition $q + 2 < 0$ is satisfied (i.e., if energy losses are determined by elastic collisions, instability does not develop). If the instability does develop, then within the framework of the model chosen, the field increases (or decreases) without limit. Upon increase the distribution function spreads out along the velocity axis.

Thus, in the nonlinear stage of Gunn instability development electron runaway occurs. The absence of characteristic velocity and time values permits us to seek self-similar solutions of Eqs. (1)-(3) in the form

$$f_0 = \varphi_1(t)\psi(v\varphi_2(t)). \tag{8}$$

Substituting Eq. (8) in normalization condition (2), we obtain

$$\varphi_1\varphi_2^{-3}\int\limits_0^\infty \xi^2\psi(\xi)\,d\xi = 1,\quad \xi = v\varphi_2(t).$$

Without limiting generality we may take

$$\int_0^\infty \xi^2\psi(\xi)\,d\xi = 1. \tag{9}$$

Then

$$\varphi_1 = \varphi_2^3. \tag{10}$$

From condition (7), using Eq. (10), we obtain

$$E = \frac{A}{C}\varphi_2^{-p}, \quad A = \frac{W_e 3m\nu_0}{e}; \tag{11}$$

$$C = -\int_0^\infty \xi^{3-p}\psi'_\xi d\xi. \tag{12}$$

We now substitute f_0 in the form of Eq. (8) in Eq. (6). Using Eqs. (10), (11), and introducing the variable ξ, after simple manipulations we have

$$\varphi_2^{-1}(\varphi_2)'_t(\xi^3\psi)'_\xi = \frac{e^2A^2}{C^2 3m^2\nu_0}\varphi_2^{2-p}\left(\xi^{2-p}\psi'_\xi\right)'_\xi + \nu_0\delta_0\varphi_2^{-p-q}\left(\xi^{3+p+q}\psi\right)'_\xi.$$

Integrating over ξ, we obtain

$$\varphi_2^{-1}(\varphi_2)'_t\,\xi^3\psi = \frac{e^2A^2}{C^2 3m^2\nu_0}\varphi_2^{2-p}\left(\xi^{2-p}\psi'_\xi\right) + \nu_0\delta_0\varphi_2^{-p-q}\xi^{3+p+q}\psi + d(t), \tag{13}$$

$$d(t) = 0, \quad \text{since} \quad f_0(v) \to 0 \ \text{for} \ v \to \infty.$$

For $q + 2 < 0$ (condition for falling dependence of W_{e_0} on E), $\varphi_2 \to 0$ as $t \to \infty$, so the second term on the right of Eq. (13) (collision integral) can be neglected, and Eq. (13) can be written as

$$\varphi_2^{p-3}(\varphi_2)'_t = \frac{e^2A^2}{C^2 3m^2\nu_0}\xi^{-p-1}\psi^{-1}\psi'_\xi = -\lambda$$

(the minus sign is introduced for convenience), whence we obtain

$$\psi(\xi) = B_1\exp\left(-\lambda\xi^{p+2}\frac{C^2 3m^2\nu_0}{e^2A^2(p+2)}\right); \tag{14}$$

$$\varphi_2(t) = (-\lambda(p-2)t + B_2)^{1/(p-2)}, \quad p \neq 2; \tag{15}$$

$$\varphi_2(t) = B_3\exp(-\lambda t), \quad p = 2. \tag{16}$$

Since $\psi(\xi)$ must satisfy condition (9), it follows that $\lambda > 0$, and from $s = 2p + q + 2 > 0$ we have $p > -(q + 2)/2$. Thus, the solution obtained is valid for p and q values as follows:

$$q + 2 < 0, \quad p > -(q+2)/2.$$

We note that the manner in which $\varphi_2(t)$ tends to zero may vary. Thus, for $p \leqslant 2$ $\varphi_2 \to 0$ $(t \to \infty)$, while for $p > 2$ $\varphi_2 \to 0$ $(t = t_0)$, i.e., we reach a zero value in a finite time $t_0 = B_2/(\lambda(p-2)$. Substituting Eqs. (15), (16) in Eq. (11), we obtain

$$E = (A/C)(\lambda(2-p)t + B_2)^{p/(2-p)}, \quad p \neq 2; \tag{17}$$

$$E = \frac{A}{C}B_3^{-p}\exp(\lambda pt), \quad p = 2. \tag{18}$$

It is evident from Eqs. (17), (18) that at $p > 2$ the field increase is discontinuous, at $p = 2$ it is exponential, and at $p < 2$ it is described by a power series.

The corresponding distribution functions have the form

$$f_0 = B_1(\lambda(2-p)t + B_2)^{3/(p-2)}\exp\left(-\lambda\xi^{p+2}\frac{C^2 3m^2\nu_0}{e^2A^2(p+2)}\right),$$

$$\xi = v(\lambda(2-p)t + B_2)^{\frac{1}{p-2}}, \quad p \neq 2,$$

$$f_0 = B_1 B_3^3 \exp(-3\lambda t) \exp\left(-\lambda \xi^{p+2} \frac{C^2 3m^2 \nu_0}{e^2 A^2 (p+2)}\right),$$

$$\xi = v B_3 e^{-\lambda t}, \quad p = 2.$$

It is clear from the course of the solution that in the asymptotic stage of Gunn instability development with increasing electric field inelastic collisions may be neglected, which is a consequence of the definite dependence of collision frequency on velocity. In this case, the dependence of distribution function on form of the inelastic process section formally disappears. We note that for instability development with decreasing electric field no simplified solutions are found.

Using Eqs. (9), (12), relationships between the constants λ, C, B_1, B_2, and B_3 can be found. For the case p = 2

$$\lambda = \frac{3\nu_0 W_e^2}{4} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}. \tag{19}$$

For the case $p \neq 2$ the coefficient of t can be determined by Eq. (17), if the latter is rewritten in the form

$$E = \left(\left(\frac{A}{C}\right)^{(2-p)/p} \lambda(2-p)t + \left(\frac{A}{C}\right)^{(2-p)/p} B_2\right)^{p/(2-p)},$$

$$\left(\frac{A}{C}\right)^{(2-p)/p} \lambda(2-p) = \left[\frac{3m\nu_0 W_e}{e(p+2)} \frac{\Gamma\left(\frac{3}{p+2}\right)}{\Gamma\left(\frac{5}{p+2}\right)}\right]^{(p+2)/p} (4-p^2)\frac{e}{3m^2\nu_0}. \tag{20}$$

We will consider the more restricted case of p and q values for which an exact solution can be found. Let p + q = 0. Then Eq. (13) takes on the form

$$\frac{\varphi_2^{-1}(\varphi_2)_t' - \nu_0\delta_0}{\varphi_2^{2-p}} = \frac{e^2A^2}{C^2 3m^2\nu_0} \xi^{-1-p} \frac{\psi_\xi'}{\psi} = -\lambda,$$

whence

$$\varphi_2 = \left(B_2 \exp((p-2)\nu_0\delta_0 t) + \frac{\lambda}{\nu_0\delta_0}\right)^{1/(p-2)}.$$

The expression for $\psi(\xi)$ is the same as in Eq. (14). Since p + q = 0 and q + 2 < 0, the solution is valid for p > 2. According to Eq. (11) we have

$$E = \left[\left(\frac{A}{C}\right)^{(2-p)/p} B_2 \exp((p-2)\nu_0\delta_0 t) + \frac{\lambda}{\nu_0\delta_0}\left(\frac{A}{C}\right)^{(2-p)/p}\right]^{p/(2-p)}. \tag{21}$$

From Eqs. (9), (12), we can define

$$\frac{\lambda}{\nu_0\delta_0}\left(\frac{A}{C}\right)^{(2-p)/p} = \left[\frac{3m\nu_0 W_e}{e(p+2)} \frac{\Gamma\left(\frac{3}{p+2}\right)}{\Gamma\left(\frac{5}{p+2}\right)}\right]^{(p+2)/p} \frac{e^2(p+2)}{3m^2\nu_0^2\delta_0^2}. \tag{22}$$

Both decreasing field (B_2 < 0) and increasing field (B_2 > 0) are described by Eq. (21). In the first case the field tends to zero exponentially (over long times), while in the second case the field increase is discontinuously abrupt. We note that in that case Eq. (21) transforms into Eq. (17). This can be shown by expanding the exponential term in Eq. (21) in a series in the vicinity of the breakoff point t_0, which is defined by $B_2 \exp[(p-2) \times \nu_0\delta_0 t_0] \times \lambda/(\nu_0\delta_0) = 0$.

TABLE 1

p	q	Γ, sec^{-1} [3]	$\hat{W}_e$	$\Gamma/\bar{\nu}_u$	Γ, sec^{-1}	$\Delta E/E$
					numerical calc.	
2	—3	$8{,}13 \cdot 10^7$	0,333	0,38	$1{,}15 \cdot 10^8$	0,005
2	—2,1	$7{,}97 \cdot 10^6$	0,024	0,049	$6{,}56 \cdot 10^6$	0,04
2	—2,02	$1{,}59 \cdot 10^5$	0,005	0,001	$1{,}45 \cdot 10^5$	0,16

To investigate the question of exit of an arbitrary solution to self-similarity, system (1)-(3) was solved numerically. Equation (1) was solved by an implicit technique using the drive method [5]. Boundary conditions were chosen as in [6]. In each time step iteration over field was carried out to satisfy Eq. (3). The initial state was chosen close to stationary. The perturbation was defined by specifying a value of W_e differing slightly from the stationary value. Then E and f_0 were perturbed at the initial time. All numerical calculations were performed for an initial value of $E/N = 0.03 \cdot 10^{-20}$ V·m² (where N is the number of particles per m³ of gas, equal to $2.69 \cdot 10^{25}$ m^{-3}, which corresponds to atmospheric pressure). The values of ν_0 and δ_0 were chosen such that at an energy of 1 eV the sections corresponding to the frequencies ν_m and ν_u were equal to 10^{-20} and $0.5 \cdot 10^{-23}$ m².

To verify the operation of the numerical technique chosen, a comparison was made with analytic theory in the linear state (for short time periods). The instability development increment Γ was calculated, and its value compared to the corresponding theoretical one, as determined by the expressions of [3]. Table 1 presents increment values for several p, q values. The analytic theory is valid when the conditions $|\hat{W}_e| \ll 1$ and $|\Gamma/\nu_u| \ll$ ($\bar{\nu}_u = \nu_u(\alpha)$, $\hat{W}_e = d \ln W_e / d \ln E$) are satisfied. The errors introduced by the finite values of these quantities are comparable in order of magnitude to the quantities $|\hat{W}_e|$ and $\Gamma/\bar{\nu}_u$, respectively. In order for the numerical calculation to generate the instability increment for the linear stage, it is necessary that $|\Delta E/E| \ll 1$ (where ΔE is the initial field perturbation). The quantity $|\Delta E/E|$ is the probable error in the increment calculation. Thus, it is evident from Table 1 that analytic theory and the numerical calculation are in good agreement within the limits of the errors indicated.

System (1)-(3) was also solved for long time periods. Results are presented in Figs. 1-3.

Figure 1 shows field intensity versus time for the case of increasing field at p = 2, q = −3. Since according to Eq. (18) the increase should occur exponentially, a logarithmic ordinate $\ln [(E/N) \cdot 10^{20}]$ is used for ease of comparison. Self-similar solution (18) then appears as a straight line. It is evident from Fig. 1 that the numerical solution (solid line) asymptotically approaches a straight line, the slope of which agrees well with the theoretical solution (dashed line).

Figure 2 shows a similar comparison for p = 1.5, q = −3 (power series field increase). The ordinate axis uses units $[(E/N) \cdot 10^{20}]^{1/3}$, so that self-similar solution (17) is now a straight line. The numerical solution (solid line) also approaches a straight line, the slope of which agrees well with the theoretical solution (dashed line).

Figure 3 shows calculation results for p = 2.5, q = −2.5, for the case of field decrease. The ordinate axis units are $[(E/N) \cdot 10^{20}]^{-1/5}$, and the abscissa, $\exp(0.5\nu_0\delta_0 t)$. Self-similar solution (21) will then have the form of a straight line, which intersects the ordinate axis at a point defined by Eq. (22). As is evident from Fig. 3, the numerical solution also approaches a straight line, whose intersection (●) with the ordinate coincides with the theoretical solution's (O).

Thus, homogeneous development of Gunn instability in a gaseous plasma under conditions where development is limited by establishment of the electron distribution function has been investigated. Approximate self-similar solutions have been found, to which the full solution obtained numerically tends. For the case of independence of the inelastic collision frequency from electron energy (p + q = 0) an exact self-similar solution has been found, toward which the full numerical solution also tends.

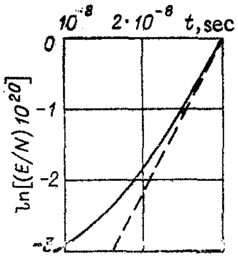

Fig. 1

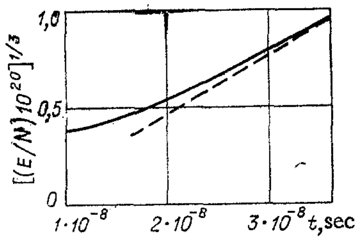

Fig. 2

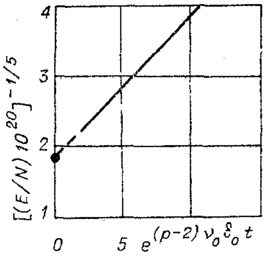

Fig. 3

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